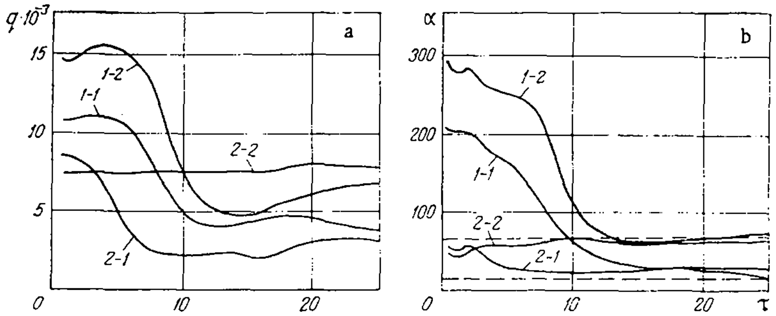

Fig. 4. Heat-flux density (a) and heat-transfer coefficient (b) vs time: the first figure corresponds to the startup conditions and the second gives the cross section.

NOTATION

M, Mach number; Re, Reynolds number; P_0, stagnation pressure; T_0, stagnation temperature; T_w, temperature of external model surface; T, temperature of internal model surface; q, unknown heat flux; τ, time; τ_m, time of process; a^*, regularization parameter; a, thermal diffusivity; λ, thermal conductivity; α, heat-transfer coefficient; R, thickness of model wall.

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NONSTEADY FLOW OF DISSOCIATING NITROGEN TETROXIDE IN A STEAM-GENERATING CHANNEL

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A one-dimensional conjugate model is proposed for the nonsteady flow of chemically reacting nitrogen tetroxide in a steam-generating channel and the boundaries of hydrodynamic stability are investigated.

1. Mathematical Model

The conjugate nonsteady problem of a flow of chemically reacting heat carrier — nitrogen tetroxide (N_2O_4) — in a steam-generating channel is investigated. In formulating the mathematical model of the two-phase flow, the steam-generating channel is considered as a system with distributed parameters, using integral characteristics such as the heat-transfer, slip, and friction coefficients, the mean flow vapor content taken over the channel cross section, and the mean-mass flow rate. The system of one-dimensional nonsteady equations describing the behavior of the induced N_2O_4 flow is complemented by the nonsteady heat-conduction equation for the channel wall.

For a more complete description of the induced two-phase flow in the channel, the channel is divided into five regions differing in conditions of motion, heat-transfer mechanisms, and the balance of dissociation in the system $N_2O_4 \rightleftharpoons 2NO_2 \rightleftharpoons 2NO + O_2$.

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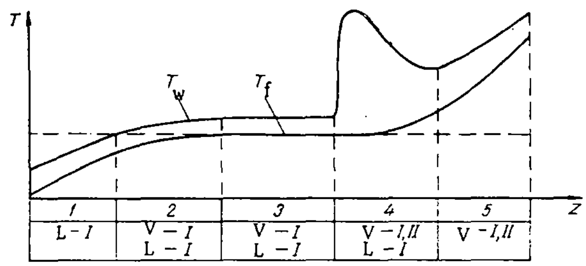

Fig. 1. Temperature distribution of flow and heat-transfer surface over the length of the channel: 1) preliminary heating of liquid; 2) surface boiling; 3) developed boiling; 4) boiling crisis of the second kind; 5) heating of vapor; I) reaction $N_2O_4 \rightleftharpoons 2NO_2$; II) $2NO_2 \rightleftharpoons 2NO + O_2$; V, vapor; L, liquid.

The characteristic distribution of the mean-integral temperatures of the flow and the heat-transfer surface within the different regions is shown in Fig. 1, together with the corresponding stages of the dissociation occurring in the liquid and vapor phases. It is assumed that $N_2O_4 \rightleftharpoons 2NO_2$ is a quasiequilibrium reaction, whereas $2NO_2 \rightleftharpoons 2NO + O_2$ is a strongly nonequilibrium reaction [1-2].

Consider the special features of the conservation equations for the various regions of the steam-generating channel.

Region 1: Preliminary Heating of Liquid. The effect of the chemical reaction in this region is taken into account by the use of effective thermophysical properties. Therefore the conservation equations are analogous to the corresponding conservation equations for chemically inert materials [2].

Region 5: Heating of Vapor. The system of nonsteady differential continuity equations of the four-component single-phase chemically reacting system $N_2O_4 \rightleftharpoons 2NO_2 \rightleftharpoons 2NO + O_2$ is of the form

$$\frac{\partial \rho_k}{\partial t} + \operatorname{div} \rho_k \vec{w}_k = \nu_k I_i. \tag{1}$$

Summing the equations for the individual components and neglecting diffusional mass transfer along the channel, the continuity equation for the flow as a whole may be obtained:

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \vec{w} = 0, \tag{2}$$

where

$$\rho = \sum_{k=1}^{4} \rho_k; \quad \rho\vec{w} = \sum_{k=1}^{4} \rho_k \vec{w}_k.$$

According to [1-2], $I_I \simeq 0$ with sufficient accuracy. Then

$$\rho_1 = \frac{m_1}{m_2 k_{cI}} \rho_2^2. \tag{3}$$

On the other hand, in the case of stoichiometric composition of the reacting mixture,

$$\rho_3 = \frac{2m_3}{m_4} \rho_4. \tag{4}$$

The system of differential equations in Eq. (2) and the continuity equation for the O_2 component of the mixture in Eq. (1), together with the algebraic relations in Eqs. (3) and (4), are completely equivalent to the initial system in Eq. (1).

The thermal-energy-conservation equation in this region is of the form

$$\frac{\partial (\rho i)}{\partial t} + \operatorname{div} (\rho i \vec{w}) = \operatorname{div} \vec{q}_h, \tag{5}$$

where

$$\rho i = \sum_{k=1}^{4} \rho_k (i_k + \Delta H_k), \quad \rho i \vec{w} = \sum_{k=1}^{4} \rho_k (i_k + \Delta H_k) \vec{w}_k.$$

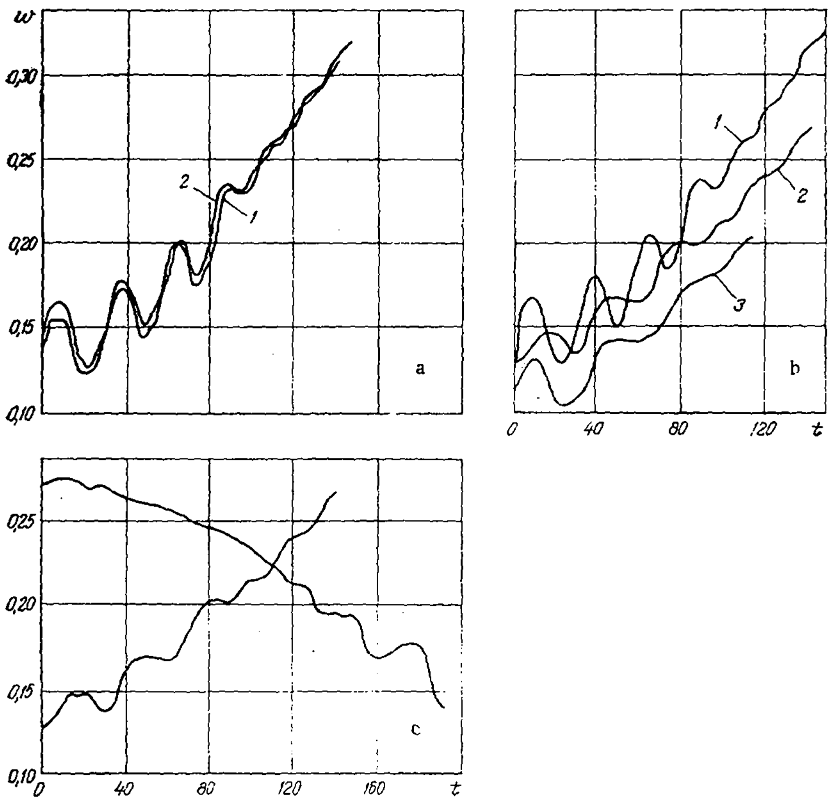

Fig. 2. Time dependence of heat-carrier velocity at channel inlet ($q = 1.16 \cdot 10^4$ W/m^2; $l = 3$ m; $d_I = 1.2 \cdot 10^{-2}$ m; w, m/sec; t, sec): a) P = 20 bar; $\Delta t = 0.1$ (1) and 0.01 (2) sec; b) P = 20 (1), 30 (2), and 40 (3) bar; c) P = 30 bar.

The derivation of Eq. (5) includes a number of assumptions which are also used below. For example, in writing the thermal balance for the channel element, no account is taken of diffusional heat transfer and heat conduction along the channel nor of dissipative scattering of energy inside the chemically reacting flow.

Region 2: Surface Boiling. The continuity equations for the liquid and vapor phases in this region are of the form

$$\frac{\partial \rho'(1-\varphi)}{\partial t} + \operatorname{div}[\rho'(1-\varphi)\vec{w}'] = I'_{sb},$$

$$\rho''_s\left(\frac{\partial\varphi}{\partial t} + \operatorname{div}(\varphi\vec{w}'')\right) = I''_{sb}. \qquad (6)$$

It is obvious that

$$I''_{sb} = -I'_{sb} = \frac{1}{r}\operatorname{div}(\vec{q}_v - \vec{q}_C). \qquad (7)$$

Integrating Eq. (6) over the channel element from cross section 1 to cross section 2 and taking Eq. (7) into account, Eq. (6) is brought to the form

$$\int_1^2 \frac{\partial}{\partial t}[\rho''_s\varphi + (1-\varphi)\rho']\,dz + [\rho''_s\varphi w'' + \rho'(1-\varphi)w']\Big|_1^2 = 0,$$

$$\int_1^2 \frac{\partial\varphi}{\partial t}\,dz + (\varphi w'')\Big|_1^2 = \left[\frac{2(z_2-z_1)}{r_I}\bar{q}_v - \frac{S_{iph}}{\pi r_I^2}\bar{q}_C\right]\frac{1}{\rho'' r}. \qquad (8)$$

The right-hand sides of these equations contain discrete-structure characteristics of the induced liquid—vapor flow in the surface-boiling region such as the number of bubbles per unit volume, the bubble diameter in the flow core, and the breakaway diameter at the heat-transfer surface. To find the breakaway diameter, the dynamics of vapor formation in the chemically reacting system is analyzed by a statistical method in which the corresponding distribution function of the vapor-forming cavities on the heated surface is determined [3].

The energy-conservation equation for the individual phases in the surface-boiling region are related and may therefore be replaced by a single thermal-energy-conservation equation for the flow as a whole:

$$\frac{\partial}{\partial t}\left[\rho_s'' i_s'' \varphi + \rho' i' (1-\varphi)\right] + \operatorname{div}\left[\rho_s'' i_s'' \varphi \vec{w}'' + \rho' i' (1-\varphi)\vec{w}'\right] = \operatorname{div}\vec{q}_{\mathrm{sb}}\,. \tag{9}$$

By analogy with the above, the momentum-conservation equation should be written separately for the liquid and vapor phases and the solution of these equations determines both the pressure gradient common to both phases and the relation between $\vec{w}''$ and $\vec{w}'$. However, within the framework of the one-dimensional model, this approach requires a knowledge of the frictional coefficients for the liquid and vapor phases not only at the channel surface but also at the phase interface. Since there has been very little investigation of friction at the phase interface, the system of differential equations is replaced by a single momentum-conservation equation for the flow as a whole and the ratio w''/w' is approximated by an algebraic relation.

In the coordinates (t, z) for the channel element from cross section 1 to cross section 2, this equation takes the form

$$\int_1^2 \frac{\partial[\rho w]_{\mathrm{M}}}{\partial t}\,dz + [\rho w^2]_{\mathrm{M}}\Big|_1^2 = \Delta P\Big|_1^2 - F\Big|_1^2 (z_2 - z_1) + [\rho]_{\mathrm{M}}\, g\,(z_2 - z_1)\cos\alpha,$$

where for the surface-boiling region

$$[\rho w]_{\mathrm{M}} = \rho_s''\varphi w'' + \rho'(1-\varphi)w', \quad [\rho]_{\mathrm{M}} = \rho_s''\varphi + \rho'(1-\varphi),$$
$$[\rho w^2]_{\mathrm{M}} = \rho_s''\varphi w''^2 + \rho'(1-\varphi)w'^2.$$

Region 3: Developed Boiling. For the developed-boiling region the system of conservation equations for the individual phases degenerates into a system of conservation equations for the flow as a whole, which is analogous to the corresponding system of conservation equations for chemically inert materials. The effect of equilibrium chemical reaction in this region is taken into account by the use of effective thermophysical properties [2].

Region 4: Boiling Crisis of the Second Kind. The system of conservation equations in this region is of the form

$$\begin{gathered}
\frac{\partial(\rho''\varphi)}{\partial t} + \operatorname{div}(\rho''\varphi\vec{w}'') = \frac{1}{r}\operatorname{div}(\vec{q}_{\mathrm{v}1} + \vec{q}_{\mathrm{v}2}),\\
\frac{\partial[\rho]_{\mathrm{M}}}{\partial t} + \operatorname{div}[\rho\vec{w}]_{\mathrm{M}} = 0,\\
\frac{\partial\rho_4}{\partial t} + \operatorname{div}(\rho_4\vec{w}_4) = m_4 I_{\mathrm{II}},\\
\frac{\partial[\rho i]_{\mathrm{M}}}{\partial t} + \operatorname{div}[\rho i\vec{w}]_{\mathrm{M}} = \operatorname{div}\vec{q}_{\mathrm{BC}2},
\end{gathered} \tag{10}$$

where

$$\rho'' = \sum_{k=1}^{4}\rho_{\mathrm{k}};\ \rho''\vec{w}'' = \sum_{k=1}^{4}\rho_{\mathrm{k}}\vec{w}_{\mathrm{k}};\ [\rho i]_{\mathrm{M}} = \rho_s' i_s'(1-\varphi) + \varphi\sum_{k=1}^{4}\rho_{\mathrm{k}}(i_{\mathrm{k}} + \Delta H_{\mathrm{k}});$$
$$[\rho i\vec{w}]_{\mathrm{M}} = \rho_s' i_s'(1-\varphi)\vec{w}' + \varphi\sum_{k=1}^{4}\rho_{\mathrm{k}}(i_{\mathrm{k}} + \Delta H_{\mathrm{k}})\vec{w}_{\mathrm{k}}.$$

The increase in the number of differential equations in Eq. (10) in comparison with Eqs. (6) and (9) is associated with the need to take into account the kinetics of the nonequilibrium chemical reaction $2NO_2 \rightleftharpoons 2NO + O_2$ in the vapor phase. It is assumed that the rate of vaporization from the drop surface in the flow core and at the heat-transfer surface is less than the rate at which the composition of the chemically reacting mixture equalizes over the channel cross section.

The temperature field in the channel cross section is described by a nonsteady heat-conduction equation

$$\frac{\partial T_C}{\partial t} = a\nabla^2 T_C + \frac{q_v}{\rho_C c_C} \tag{11}$$

with the following boundary conditions

$$\lambda_C \frac{\partial T_C}{\partial r}\Big|_{r=r_I} = -q_w.$$

Using the integral Laplace transform

$$\bar{f} = \int_0^\infty f(t)\exp(-st)\,dt$$

and the method of partial similarity [4], i.e., introducing the transfer function $\bar{\vartheta}$, Eq. (11) takes the form

$$s\delta T_C = \bar{\vartheta}\delta T_w + \frac{\delta q_v}{\rho_C c_C}(1-\bar{\vartheta}). \tag{12}$$

The expression for $\bar{\vartheta}$ is taken from [4]

$$\bar{\vartheta} = \frac{\operatorname{th}\sqrt{3s\tau_i}}{\sqrt{3s\tau_i}}.$$

Reducing the transfer function to a form convenient for the subsequent manipulations [5], and passing from the integral transform to the original, the result is

$$\frac{\partial T_w}{\partial t} = -\frac{q_v}{\rho_C c_C} + a_1\frac{\partial T_C}{\partial t} + \frac{\bar{T}_C - T_w}{a_2\tau_i}. \tag{13}$$

In addition, the thermal-balance equation is

$$\alpha F_w(T_w - T_f) = Q - \rho_C c_C V_C \frac{\partial \bar{T}_C}{\partial t}, \tag{14}$$

where $\bar{T}_C$ is the mean channel-wall temperature, taken over the cross section.

The system in equations (13) and (14), together with all the above equations and auxiliary relations, completely describes the behavior of the nonsteady flow of dissociating nitrogen tetroxide in the steam-generating channel. The heat-transfer, friction, and slip coefficients required within the framework of the one-dimensional model are determined from equations given in [6-8], [7, 9, 10], and [11-13], respectively.

II. Stability of Motion of Dissociation Flow in a Steam-Generating Channel

For certain combinations of dynamic, thermal, and physical characteristics of the induced flow and of geometric and physical characteristics of the channel, flow-rate oscillations occur in the channel and also oscillations of the channel-wall and flow temperatures, shift in the boundaries of the two-phase region, and even — in the case of resonance phenomena — transfer of material from one channel to another.

Oscillatory stability of the induced two-phase flow is a consequence both of the discrete flow structure and the distinctly periodic vaporization processes in the surface-boiling region and of the nonuniformity of the hydrodynamic characteristics along the channel axis.

However, in the present state of hydrodynamic theory, it is not possible to write a mathematically rigorous description of all aspects of the appearance of hydrodynamic instability; therefore, in developing a particular mathematical model it is necessary to make a number of simplifying assumptions.

The use of the given mathematical model assumes, in particular, that a one-dimensional model of induced flow is sufficient for the description of hydrodynamic instability.

A program for the calculation of the dynamics of a chemically reacting N_2O_4 flow has been realized on a Minsk-32 computer. The differential equations are solved by a modified Euler method; the modification is that in some cases (for example, in solving the equations for the mean-integral channel-wall temperature and the temperature of the heat-transfer surface) a semianalytic solution of the initial equations is chosen as the zero approximation.

The numerical stability of the final result was verified by numerical experiments on the computer. One of the results of the verification is given in Fig. 2a, which shows the time dependence of the flow rate at the channel inlet. Curve 1 corresponds to $\Delta t = 0.1$ sec and curve 2 to $\Delta t = 0.01$ sec.

The search for regions of hydrodynamic instability was made by specifying a nonsteady pressure drop over the whole channel. The transition from a region of stability (instability) to a region of instability (stability) is fixed by the development (quenching) of heat-transfer-agent flow-rate oscillations at the channel inlet. In Fig. 2b, typical curves of the heat-transfer-agent velocity at the inlet against the time are shown; there is a clear tendency for the boundary of the region of hydrodynamic instability to move downward with increasing pressure.

The correspondence between the boundaries of the hydrodynamic-instability region observed on approach from regions of increased and diminished flow rate is shown in Fig. 2c. Some delay in the development of quenching of flow-rate oscillations is evidently associated with the presence of relaxational processes when the system passes from one state to another.

The results of applying the mathematical model here described to hydrodynamic instability give a sufficient indication of its effectiveness for the study of various characteristics of an unsteady chemically reacting flow of N_2O_4.

NOTATION

t, time; z, longitudinal coordinate; r, radius; d, diameter; V, volume; l, channel length; T, temperature; ρ, density; i, enthalpy; P, pressure; m, molecular weight; r, latent heat of vaporization; c, specific heat; a, thermal diffusivity, constant; w, true phase velocity; I, chemical reaction rate, specific source of mass; ν, product of molecular weight and stoichiometric number; k_C, rate constant of chemical reaction; q, specific heat flux; Q, heat flux; φ, mean volumetric vapor content of flow; g, acceleration due to gravity; ΔP, pressure difference; F, pressure losses due to friction, surface; H, binding energy; S, surface; τ_i, thermal-lag time; Δt, time increment of numerical integration; α, angle between vectors $\vec{g}$ and $\vec{w}$. Indices: i = I, II, reactions $N_2O_4 \rightleftharpoons 2NO_2$ and $2NO_2 \rightleftharpoons 2NO + O_2$; k = 1, 2, 3, 4, mixture components (1, N_2O_4; 2, NO_2; 3, NO; 4, O_2); C, channel; s, saturation; sb, surface boiling; v, vaporization; BC2, boiling crisis of second kind; h, heating; f, flow; v1, v2, vaporization at the wall and in the flow core; w, wall; M, mixture; v, volume; I, internal; iph, interphase. A prime denotes the liquid phase and a double prime the vapor phase.

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USE OF GENERALIZED DIFFUSION COEFFICIENTS IN SOLVING CONJUGATE PROBLEMS

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UDC 536.24

A numerical method is used to solve the conjugate problem of the heating of a graphite body in a high-temperature gas flow.

Calculation of the heating and loss of thermoprotective material when high-temperature gas (air, carbon dioxide, etc.) flows past an eroding surface involves the solution of a system of differential boundary-layer equations and the nonsteady heat-conduction equation for a solid. Consider the flow of a chemically reacting mixture in the vicinity of the forward critical point of a graphite body (Fig. 1). Steady laminar flow of thin mixture (consisting of ν elements and N components) is described by the following system of differential equations [1]: the continuity equation for the mixture

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial y}(\rho v r) = 0; \tag{1}$$

the momentum equation

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right); \tag{2}$$

the diffusion equation for a chemical element

$$\rho u \frac{\partial \bar{c}_\tau}{\partial x} + \rho v \frac{\partial \bar{c}_\tau}{\partial y} + \frac{\partial \bar{K}_\tau}{\partial y} = 0; \tag{3}$$

$$(\tau = 1, 2, \ldots, \nu - 1),$$

where

$$\bar{c}_\tau = \sum_{i=1}^{N} n_{\tau i} M_\tau c_i / M_i; \quad \bar{K}_\tau = \sum_{i=1}^{N} n_{\tau i} M_\tau K_i / M_i;$$

the equation of thermochemical equilibrium for a reaction of the type

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